

Contents

	<i>Preface</i>	<i>page xv</i>
Part 1	Mathematical tools	1
M1	Algebra of vectors	3
M1.1	Basic concepts and definitions	3
M1.2	Reference frames	6
M1.3	Vector multiplication	7
M1.4	Reciprocal coordinate systems	15
M1.5	Vector representations	19
M1.6	Products of vectors in general coordinate systems	22
M1.7	Problems	23
M2	Vector functions	25
M2.1	Basic definitions and operations	25
M2.2	Special dyadics	28
M2.3	Principal-axis transformation of symmetric tensors	32
M2.4	Invariants of a dyadic	34
M2.5	Tensor algebra	40
M2.6	Problems	42
M3	Differential relations	43
M3.1	Differentiation of extensive functions	43
M3.2	The Hamilton operator in generalized coordinate systems	48
M3.3	The spatial derivative of the basis vectors	51
M3.4	Differential invariants in generalized coordinate systems	53
M3.5	Additional applications	56
M3.6	Problems	60
M4	Coordinate transformations	62
M4.1	Transformation relations of time-independent coordinate systems	62

M4.2	Transformation relations of time-dependent coordinate systems	67
M4.3	Problems	73
M5	The method of covariant differentiation	75
M5.1	Spatial differentiation of vectors and dyadics	75
M5.2	Time differentiation of vectors and dyadics	79
M5.3	The local dyadic of v_P	82
M5.4	Problems	83
M6	Integral operations	84
M6.1	Curves, surfaces, and volumes in the general q^i system	84
M6.2	Line integrals, surface integrals, and volume integrals	87
M6.3	Integral theorems	90
M6.4	Fluid lines, surfaces, and volumes	94
M6.5	Time differentiation of fluid integrals	96
M6.6	The general form of the budget equation	101
M6.7	Gauss' theorem and the Dirac delta function	104
M6.8	Solution of Poisson's differential equation	106
M6.9	Appendix: Remarks on Euclidian and Riemannian spaces	107
M6.10	Problems	110
M7	Introduction to the concepts of nonlinear dynamics	111
M7.1	One-dimensional flow	111
M7.2	Two-dimensional flow	116
Part 2	Dynamics of the atmosphere	131
1	The laws of atmospheric motion	133
1.1	The equation of absolute motion	133
1.2	The energy budget in the absolute reference system	136
1.3	The geographical coordinate system	137
1.4	The equation of relative motion	146
1.5	The energy budget of the general relative system	147
1.6	The decomposition of the equation of motion	150
1.7	Problems	154
2	Scale analysis	157
2.1	An outline of the method	157
2.2	Practical formulation of the dimensionless flow numbers	159
2.3	Scale analysis of large-scale frictionless motion	161
2.4	The geostrophic wind and the Euler wind	167
2.5	The equation of motion on a tangential plane	169
2.6	Problems	169

3	The material and the local description of flow	171
3.1	The description of Lagrange	171
3.2	Lagrange's version of the continuity equation	173
3.3	An example of the use of Lagrangian coordinates	175
3.4	The local description of Euler	182
3.5	Transformation from the Eulerian to the Lagrangian system	186
3.6	Problems	187
4	Atmospheric flow fields	189
4.1	The velocity dyadic	189
4.2	The deformation of the continuum	193
4.3	Individual changes with time of geometric fluid configurations	199
4.4	Problems	205
5	The Navier–Stokes stress tensor	206
5.1	The general stress tensor	206
5.2	Equilibrium conditions in the stress field	208
5.3	Symmetry of the stress tensor	209
5.4	The frictional stress tensor and the deformation dyadic	210
5.5	Problems	212
6	The Helmholtz theorem	214
6.1	The three-dimensional Helmholtz theorem	214
6.2	The two-dimensional Helmholtz theorem	216
6.3	Problems	217
7	Kinematics of two-dimensional flow	218
7.1	Atmospheric flow fields	218
7.2	Two-dimensional streamlines and normals	222
7.3	Streamlines in a drifting coordinate system	225
7.4	Problems	228
8	Natural coordinates	230
8.1	Introduction	230
8.2	Differential definitions of the coordinate lines	232
8.3	Metric relationships	235
8.4	Blaton's equation	236
8.5	Individual and local time derivatives of the velocity	238
8.6	Differential invariants	239
8.7	The equation of motion for frictionless horizontal flow	242
8.8	The gradient wind relation	243
8.9	Problems	244

9	Boundary surfaces and boundary conditions	246
9.1	Introduction	246
9.2	Differential operations at discontinuity surfaces	247
9.3	Particle invariance at boundary surfaces, displacement velocities	251
9.4	The kinematic boundary-surface condition	253
9.5	The dynamic boundary-surface condition	258
9.6	The zeroth-order discontinuity surface	259
9.7	An example of a first-order discontinuity surface	265
9.8	Problems	267
10	Circulation and vorticity theorems	268
10.1	Ertel's form of the continuity equation	268
10.2	The baroclinic Weber transformation	271
10.3	The baroclinic Ertel–Rossby invariant	275
10.4	Circulation and vorticity theorems for frictionless baroclinic flow	276
10.5	Circulation and vorticity theorems for frictionless barotropic flow	293
10.6	Problems	301
11	Turbulent systems	302
11.1	Simple averages and fluctuations	302
11.2	Weighted averages and fluctuations	304
11.3	Averaging the individual time derivative and the budget operator	306
11.4	Integral means	307
11.5	Budget equations of the turbulent system	310
11.6	The energy budget of the turbulent system	313
11.7	Diagnostic and prognostic equations of turbulent systems	315
11.8	Production of entropy in the microturbulent system	319
11.9	Problems	324
12	An excursion into spectral turbulence theory	326
12.1	Fourier Representation of the continuity equation and the equation of motion	326
12.2	The budget equation for the amplitude of the kinetic energy	331
12.3	Isotropic conditions, the transition to the continuous wavenumber space	333
12.4	The Heisenberg spectrum	336
12.5	Relations for the Heisenberg exchange coefficient	340
12.6	A prognostic equation for the exchange coefficient	341

12.7	Concluding remarks on closure procedures	346
12.8	Problems	348
13	The atmospheric boundary layer	349
13.1	Introduction	349
13.2	Prandtl-layer theory	350
13.3	The Monin–Obukhov similarity theory of the neutral Prandtl layer	358
13.4	The Monin–Obukhov similarity theory of the diabatic Prandtl layer	362
13.5	Application of the Prandtl-layer theory in numerical prognostic models	369
13.6	The fluxes, the dissipation of energy, and the exchange coefficients	371
13.7	The interface condition at the earth's surface	372
13.8	The Ekman layer – the classical approach	375
13.9	The composite Ekman layer	381
13.10	Ekman pumping	388
13.11	Appendix A: Dimensional analysis	391
13.12	Appendix B: The mixing length	394
13.13	Problems	396
14	Wave motion in the atmosphere	398
14.1	The representation of waves	398
14.2	The group velocity	401
14.3	Perturbation theory	403
14.4	Pure sound waves	407
14.5	Sound waves and gravity waves	410
14.6	Lamb waves	418
14.7	Lee waves	418
14.8	Propagation of energy	418
14.9	External gravity waves	422
14.10	Internal gravity waves	426
14.11	Nonlinear waves in the atmosphere	431
14.12	Problems	434
15	The barotropic model	435
15.1	The basic assumptions of the barotropic model	435
15.2	The unfiltered barotropic prediction model	437
15.3	The filtered barotropic model	450
15.4	Barotropic instability	452
15.5	The mechanism of barotropic development	463
15.6	Appendix	468
15.7	Problems	470

16	Rossby waves	471
16.1	One- and two-dimensional Rossby waves	471
16.2	Three-dimensional Rossby waves	476
16.3	Normal-mode considerations	479
16.4	Energy transport by Rossby waves	482
16.5	The influence of friction on the stationary Rossby wave	483
16.6	Barotropic equatorial waves	484
16.7	The principle of geostrophic adjustment	487
16.8	Appendix	493
16.9	Problems	494
17	Inertial and dynamic stability	495
17.1	Inertial motion in a horizontally homogeneous pressure field	495
17.2	Inertial motion in a homogeneous geostrophic wind field	497
17.3	Inertial motion in a geostrophic shear wind field	498
17.4	Derivation of the stability criteria in the geostrophic wind field	501
17.5	Sectorial stability and instability	504
17.6	Sectorial stability for normal atmospheric conditions	509
17.7	Sectorial stability and instability with permanent adaptation	510
17.8	Problems	512
18	The equation of motion in general coordinate systems	513
18.1	Introduction	513
18.2	The covariant equation of motion in general coordinate systems	514
18.3	The contravariant equation of motion in general coordinate systems	518
18.4	The equation of motion in orthogonal coordinate systems	520
18.5	Lagrange's equation of motion	523
18.6	Hamilton's equation of motion	527
18.7	Appendix	530
18.8	Problems	531
19	The geographical coordinate system	532
19.1	The equation of motion	532
19.2	Application of Lagrange's equation of motion	536
19.3	The first metric simplification	538
19.4	The coordinate simplification	539
19.5	The continuity equation	540
19.6	Problems	541

20	The stereographic coordinate system	542
20.1	The stereographic projection	542
20.2	Metric forms in stereographic coordinates	546
20.3	The absolute kinetic energy in stereographic coordinates	549
20.4	The equation of motion in the stereographic Cartesian coordinates	550
20.5	The equation of motion in stereographic cylindrical coordinates	554
20.6	The continuity equation	556
20.7	The equation of motion on the tangential plane	558
20.8	The equation of motion in Lagrangian enumeration coordinates	559
20.9	Problems	564
21	Orography-following coordinate systems	565
21.1	The metric of the η system	565
21.2	The equation of motion in the η system	568
21.3	The continuity equation in the η system	571
21.4	Problems	571
22	The stereographic system with a generalized vertical coordinate	572
22.1	The ξ transformation and resulting equations	573
22.2	The pressure system	577
22.3	The solution scheme using the pressure system	579
22.4	The solution to a simplified prediction problem	582
22.5	The solution scheme with a normalized pressure coordinate	584
22.6	The solution scheme with potential temperature as vertical coordinate	587
22.7	Problems	589
23	A quasi-geostrophic baroclinic model	591
23.1	Introduction	591
23.2	The first law of thermodynamics in various forms	592
23.4	The vorticity and the divergence equation	593
23.5	The first and second filter conditions	595
23.6	The geostrophic approximation of the heat equation	597
23.7	The geostrophic approximation of the vorticity equation	603
23.8	The ω equation	605
23.9	The Philipps approximation of the ageostrophic component of the horizontal wind	609
23.10	Applications of the Philipps wind	614
23.11	Problems	617

24	A two-level prognostic model, baroclinic instability	619
24.1	Introduction	619
24.2	The mathematical development of the two-level model	619
24.3	The Phillips quasi-geostrophic two-level circulation model	623
24.4	Baroclinic instability	624
24.5	Problems	633
25	An excursion concerning numerical procedures	634
25.1	Numerical stability of the one-dimensional advection equation	634
25.2	Application of forward-in-time and central-in-space difference quotients	640
25.3	A practical method for the elimination of the weak instability	642
25.4	The implicit method	642
25.5	The aliasing error and nonlinear instability	645
25.6	Problems	648
26	Modeling of atmospheric flow by spectral techniques	649
26.1	Introduction	649
26.2	The basic equations	650
26.3	Horizontal discretization	655
26.4	Problems	667
27	Predictability	669
27.1	Derivation and discussion of the Lorenz equations	669
27.2	The effect of uncertainties in the initial conditions	681
27.3	Limitations of deterministic predictability of the atmosphere	683
27.4	Basic equations of the approximate stochastic dynamic method	689
27.5	Problems	692
	Answers to Problems	693
	List of frequently used symbols	702
	References and bibliography	706