

Contents

Preface ix

1 Introduction to Differential Equations 1

- 1.1** Differential Equation Models 2
- 1.2** The Derivative 7
- 1.3** Integration 10

2 First-Order Equations 18

- 2.1** Differential Equations and Solutions 18
- 2.2** Solutions to Separable Equations 31
- 2.3** Models of Motion 44
- 2.4** Linear Equations 54
- 2.5** Mixing Problems 64
- 2.6** Exact Differential Equations 73
- 2.7** Existence and Uniqueness of Solutions 90
- 2.8** Dependence of Solutions on Initial Conditions 102
- 2.9** Autonomous Equations and Stability 107
- Project 2.10 The Daredevil Skydiver** 120

3 Modeling and Applications 123

- 3.1** Modeling Population Growth 124
- 3.2** Models and the Real World 138
- 3.3** Personal Finance 143
- 3.4** Electrical Circuits 152
- Project 3.5 The Spruce Budworm** 158
- Project 3.6 Social Security, Now or Later** 161

4 Second-Order Equations 163

- 4.1** Definitions and Examples 163
- 4.2** Second-Order Equations and Systems 174
- 4.3** Linear, Homogeneous Equations with Constant Coefficients 179
- 4.4** Harmonic Motion 190

4.5	Inhomogeneous Equations; the Method of Undetermined Coefficients	199
4.6	Variation of Parameters	209
4.7	Forced Harmonic Motion	215
Project 4.8 Nonlinear Oscillators		228

5 The Laplace Transform 231

5.1	The Definition of the Laplace Transform	232
5.2	Basic Properties of the Laplace Transform	241
5.3	The Inverse Laplace Transform	248
5.4	Using the Laplace Transform to Solve Differential Equations	256
5.5	Discontinuous Forcing Terms	266
5.6	The Delta Function	280
5.7	Convolutions	287
5.8	Summary	298
Project 5.9 Forced Harmonic Oscillators		299

6 Numerical Methods 301

6.1	Euler's Method	302
6.2	Runge-Kutta Methods	314
6.3	Numerical Error Comparisons	322
6.4	Practical Use of Solvers	327
6.5	A Cautionary Tale	332
Project 6.6 Numerical Error Comparison		334

7 Matrix Algebra 335

7.1	Vectors and Matrices	335
7.2	The Geometry of Systems of Linear Equations	348
7.3	Solving Systems of Equations	353
7.4	Properties of Solution Sets	362
7.5	Subspaces	371
7.6	Determinants	384

8 An Introduction to Systems 396

8.1	Definitions and Examples	396
8.2	Geometric Interpretation of Solutions	405
8.3	Qualitative Analysis	417
8.4	Linear Systems	425
Project 8.5 Long-Term Behavior of Solutions		441

9 Linear Systems with Constant Coefficients 444

- 9.1 Overview of the Technique 444
- 9.2 Planar Systems 452
- 9.3 Phase Plane Portraits 466
- 9.4 Higher Dimensional Systems 484
- 9.5 The Exponential of a Matrix 492
- 9.6 Qualitative Analysis of Linear Systems 510
- 9.7 Higher-Order Linear Equations 515
- 9.8 Inhomogeneous Linear Systems 528
- Project 9.9** Phase Plane Portraits 538
- Project 9.10** Oscillations of Linear Molecules 539

10 Nonlinear Systems 545

- 10.1 The Linearization of a Nonlinear System 545
- 10.2 Long-Term Behavior of Solutions 559
- 10.3 Invariant Sets and the Use of Nullclines 566
- 10.4 Long-Term Behavior of Solutions to Planar Systems 574
- 10.5 Conserved Quantities 586
- 10.6 Nonlinear Mechanics 592
- 10.7 The Method of Lyapunov 609
- 10.8 Predator–Prey Systems 619
- Project 10.9** Human Immune Response to Infectious Disease 631
- Project 10.10** Analysis of Competing Species 634

11 Series Solutions to Differential Equations 637

- 11.1 Review of Power Series 638
- 11.2 Series Solutions Near Ordinary Points 650
- 11.3 Legendre’s Equation 662
- 11.4 Types of Singular Points—Euler’s Equation 668
- 11.5 Series Solutions Near Regular Singular Points 677
- 11.6 Solutions in the Exceptional Cases 690
- 11.7 Bessel’s Equation and Bessel Functions 701

12 Fourier Series 712

- 12.1 Computation of Fourier Series 713
- 12.2 Convergence of Fourier Series 724
- 12.3 Fourier Cosine and Sine Series 733
- 12.4 The Complex Form of a Fourier Series 738
- 12.5 The Discrete Fourier Transform and the FFT 741

13 Partial Differential Equations 750

13.1	Derivation of the Heat Equation	750
13.2	Separation of Variables for the Heat Equation	756
13.3	The Wave Equation	768
13.4	Laplace's Equation	778
13.5	Laplace's Equation on a Disk	785
13.6	Sturm Liouville Problems	791
13.7	Orthogonality and Generalized Fourier Series	801
13.8	Temperature in a Ball—Legendre Polynomials	810
13.9	Time Dependent PDEs in Higher Dimension	814
13.10	Domains with Circular Symmetry—Bessel Functions	821

Answers to Odd-Numbered Problems A-1

Index I-1